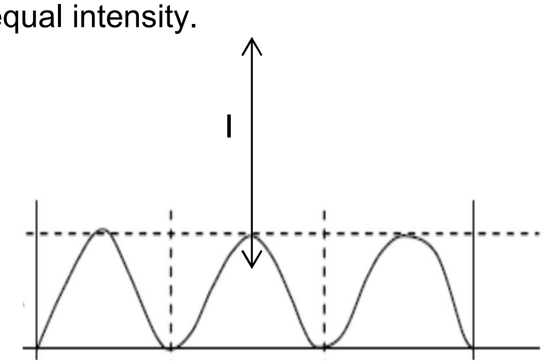
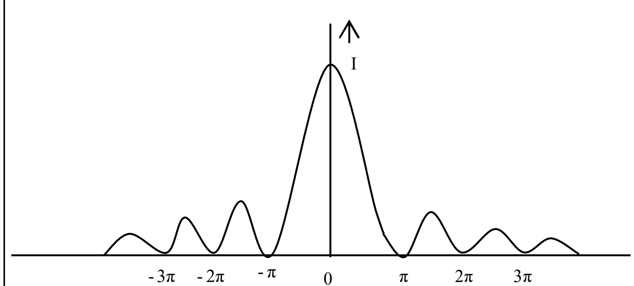
MODULE –II (OPTICS)

Diffraction

The phenomenon which demonstrates the bending of light around an obstacle is known as diffraction. The diffraction phenomenon was correctly interpreted by “Fresnel”. He combined the Huygens principle of secondary wavelets with principle of interference and concluded that the diffraction occurs due to the interference of secondary wavelets, originating from various points of the wave front which are not obstructed by the obstacle.

Difference between interference & Diffraction

Interference	Diffraction
(1) Interference phenomenon occurs between two waves originating from two coherent sources.	(1) Diffraction phenomenon occurs between secondary wavelets originating from unobstructed portion of the incident wave-front.
(2) The dark fringe of the interference fringe is almost perfectly dark.	(2) The dark portion of the diffraction band is not perfectly dark.
(3) In interference pattern the widths of the interference fringes may be uniform (Biprism) or may not be uniform (Newton’s rings)	(3) In diffraction pattern the width of the diffraction bands are never equal.
(4) In interference pattern maxima are of equal intensity. 	(4) In diffraction pattern maxima are of unequal intensity. 

Classification of diffraction

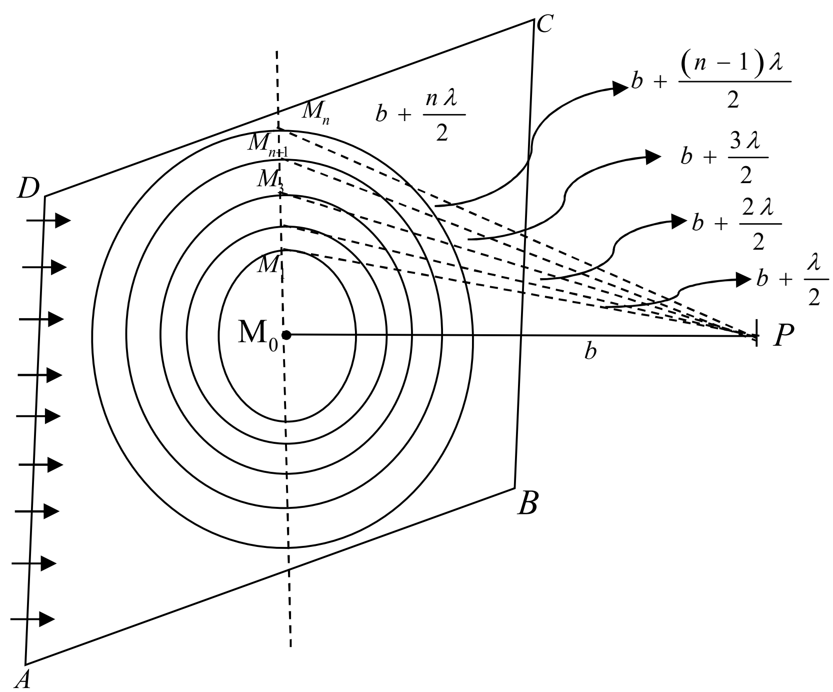
The diffraction phenomenon is broadly classified into following two categories.

Fresnel's diffraction	Fraunhofer's diffraction
1. In this class of diffraction, light source and the obstacle are separated by finite distance	1. In this class of diffraction light source and the obstacle are separated by infinite distance
2. The wave front incident on the obstacle is either cylindrical or spherical.	2. The wave front incident on the obstacle is plane wave.
3. Example – Zone plate, diffraction at a straight edge.	3. Example – Diffraction due to single slit, double slit and diffraction grating.

N.B.: In laboratory infinite distance is created by using a biconvex lens between the source and obstacle. This lens produces parallel beam of light, which means that the light source is present at infinity.

Fresnel's half period zone

Fresnel's approach to the study of diffraction divide the wave front in to small zones called half-period zones and the combined effect of such zones at a point ahead of the wave front is called diffraction. The wave front considered is either spherical or cylindrical but for simplicity we have considered a plane wave front.



Let,

ABCD = Plane wave front propagating in z- direction.

'*P*' = Point, at which the diffraction pattern has to be obtained.

PM_0 =Perpendicular drawn from P on wave front ABCD.

Taking 'P' as center, let us draw concentric spheres of radius PM_0, PM_1, PM_2………. PM_n

From the figure,

$PM_0 = b$ $PM_1 = b + \lambda/2$ $PM_2 = b + 2\lambda/2 = b + \lambda$ and so on.

Now, $PM_1 – PM_0 = \lambda/2$, $PM_2 - PM_1 = \lambda/2$,………. $PMn – PM_{n-1} = \lambda/2$

So, the path difference between the waves originating from the circumference of the circles is $\lambda/2$ and hence the phase difference $\varphi = \frac{2\pi}{\lambda} . \frac{\lambda}{2} = \pi$. A phase difference of π correspond to half period that is *T/2*. So, the circular area enclosed by the circle 'M_1' is called Fresnel's first half period zone. The circular strip enclosed by the circles M_1&M_2 is called Fresnel's second half period zone and so on. Thus, the constructions divide the entire wave front in to a large number of half period zones called Fresnel's half period zones.

So, the annular region between the adjacent circles is called half period zone and waves coming out of each zone interfere to produce diffraction.

The amplitude at point *P* depends upon,

(a) Area of the Zone (b) Average Distance and (c) Obliquity Factor

(a) Area of the Zone:

From the figure, the area of the first half period zone is,

$$\pi(M_0M_1)^2 = \pi[(PM_1)^2 - (PM_0)^2]$$

$$= \pi\left[\left(b + \lambda/2\right)^2 - (b)^2\right]$$

$$= \pi\, b\lambda \text{ --------- } (1) \qquad \text{[Simplifying and neglecting higher powers of } \lambda \text{]}$$

Now, $\pi(M_0M_1)^2 = \pi\, b\lambda \Rightarrow \pi r_1^2 = \pi\, b\lambda \quad \Rightarrow r_1 = \sqrt{b\lambda}$

Again, Radius of 2nd half period zone is,

$$OM_2 = \left[(M_2P)^2 - (OP)^2\right]^{\frac{1}{2}} \quad \Rightarrow \quad OM_2 = \left[(b+\lambda)^2 - b^2\right]^{\frac{1}{2}}$$

$$\Rightarrow \quad OM_2 = \sqrt{2b\lambda} = r_2$$

Similarly, $r_3 = \sqrt{3b\lambda}$

$r_n = \sqrt{nb\lambda}$ n = 1, 2, 3, 4, ...

Area of the 2nd half period zone is, (Annular space between two circles)

$$\pi r_2^{\,2} - \pi r_1^{\,2} = \pi\left(2b\lambda - b\lambda\right) = \pi b\lambda$$

Similarly, on calculation it can be shown that area of each half period zone is $\pi b\lambda$. So, it can be concluded that the area of all the half period zones are same.

(b) Average Distance:

The distance of inner boundary of nth zone from the point P is $PM_{n-1} = \left[b + (n-1)\frac{\lambda}{2}\right]$

The distance from the outer boundary of nth zone is $PM_n = \left[b + \frac{n\lambda}{2}\right]$

The average distance of nth zone from P is,

$$\frac{PM_n + PM_{n-1}}{2} = \frac{\left[b + (n-1)\frac{\lambda}{2}\right] + \left[b + \frac{n\lambda}{2}\right]}{2}$$

$$= \frac{2b + (2n-1)\frac{\lambda}{2}}{2} = \frac{4b + (2n-1)\lambda}{4} = b + (2n-1)\frac{\lambda}{4}$$

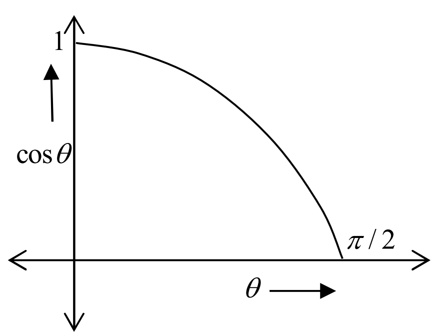
(c) Obliquity Factor:

The amplitude in terms obliquity factor can be written as,

$$A_n \propto (1 + \cos\theta_n)$$

For $\theta = \frac{\pi}{2}$, $A_n \propto 1 \;\; as \quad \cos\frac{\pi}{2} = 0$

$\theta = 0, \quad A_n \propto 2$



Now, the resultant amplitude at point P can be considered as follows

1. Contribution to amplitude due to the area is same for all zones.
2. Contribution due to average distance shows that the distance of the zone increases with the order of the zones. So, the contribution to the amplitude is more for lower order zones and less for higher order zones.
3. The value of $\cos\theta$ decreases more rapidly as we move from 0 to $\frac{\pi}{2}$. So, the amplitude decreases continuously due to the obliquity factor.

Amplitude at point 'P'

Let, $A_1, A_2, A_3 ....A_n$ be the amplitudes of secondary wavelets at point P due to 1st, 2nd, 3rd ,…..nth half period zones.

Since the phase difference between any two consecutive half period zone is opposite. So, the resultant amplitude will be,

$$A = A_1 - A_2 + A_3 - A_4 + ...+(-1)^{n-1} A_n$$

As the amplitudes are decreasing steadily, the resultant amplitude at point P due to any zone can be approximately taken as the arithmetic mean of the preceding and succeeding zones.

So, $A_n = \dfrac{A_{n-1} + A_{n+1}}{2}$

For n odd,

$$A_{odd} = \frac{A_1}{2} + \left(\frac{A_1}{2} - A_2 + \frac{A_3}{2}\right) + \left(\frac{A_3}{2} - A_4 + \frac{A_5}{2}\right) + ...\frac{A_n}{2}$$

$$\Rightarrow \qquad A_{odd} = \frac{A_1}{2} + \frac{A_n}{2}$$

Similarly, for *n* even, $A_{even} = \dfrac{A_1}{2} - \dfrac{A_n}{2}$

For large value of *n*, $n \to \infty, \quad A_n \to 0$

$$\Rightarrow \quad \boxed{A = \frac{A_1}{2}}$$

From the above it can be concluded that, the illumination is primarily due to the 1st half period zone. There is very less contribution due to few lower order zones.

Zone plate

The qualitative oppose of Fresnel's diffraction in half period zones was tested on a device called zone plate.

Construction

On the sheet of plane paper large no. of concentric circles with radii proportional to the square root of natural numbers are drawn. These radii are the radii of Fresnel's half period zones.

If the inner most circle represents the 1st half period zone, the annular space between two successive circles represent the subsequent zones.

The alternate zones are painted black and a reduced photograph is taken on a plane glass plate. The negative of the photograph is called as zone plate.

In the negative the original white appears as black and is called the opaque surface while the original black appears as white called the transparent surface.

Depending upon the blackening i.e., either even numbered annular space are blackened or odd numbered annular spaces are blackened, two types of zone plates are constructed.

Positive zone Plate: -

When the central circular zone is transparent along with the other odd no. zones, then the zone plate is called as +ve zone plate.

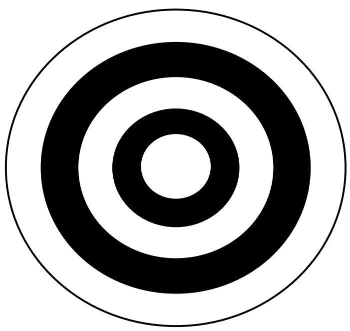


fig :1 positive zone plate

Negative Zone plate: -

When the central circular zone is opaque along with the other even no. zones, then the zone plate is called as -ve zone plate.

Here the even no zone act as transparent surface.

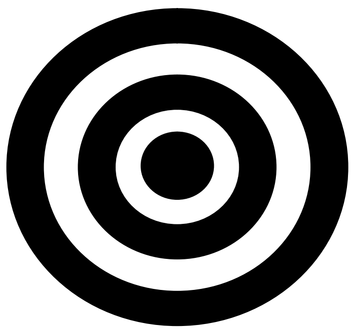


fig :2 negative zone plate

Action of zone plate

In the diagram,

S = Source

P = Point of observation

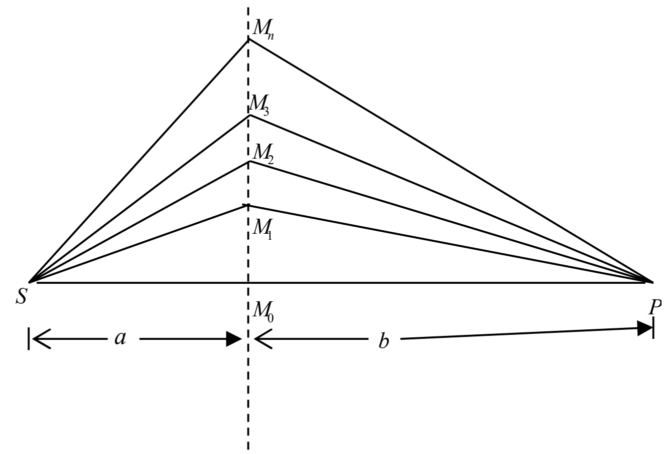
$SM_0 = a$ = object distance

$M_0P = b$ = Image distance

SM_1P = First half period zone

$M_0M_1 = r_1$ = Radius of first half period zone

Similarly, $M_0M_2 = r_2$, $M_0M_3 = r_3$ and so on.



Let us consider a positive zone plate. The even no zones are opaque and odd no. zones are transparent so the intensity at point p will be due to the secondary wavelets coming from the odd number zones. So, the phase difference between the arrivals from each neighboring pair will be $\pi + \pi = 2\pi$. Hence the amplitude of P will be $A = A_1 + A_3 + A_5 + ...$

The amplitude due to the unobstructed wave front is $\frac{A_1}{2}$.

So. by obstructing the wave front by a zone plate there will concentration of light and hence the intensity at point P increases. This means that a zone plate acts like a converging lens. Hence the equivalent focal length can be calculated as follows.

Let us consider the 1st zone.

$$SM_1P - SM_0P = \frac{\lambda}{2}$$

$$\Rightarrow \quad (SM_1 + M_1P) - (SM_0 + M_0P) = \frac{\lambda}{2}$$

$$\Rightarrow \quad SM_1 + M_1P = a + b + \frac{\lambda}{2}$$

$$\Rightarrow \quad \left[(SM_0)^2 + (M_0M_1)^2\right]^{\frac{1}{2}} + \left[(M_0P)^2 + (M_0M_1)^2\right]^{\frac{1}{2}} = a + b + \frac{\lambda}{2}$$

$$\Rightarrow \quad \left(a^2 + r_1^2\right)^{\frac{1}{2}} + \left(b^2 + r_1^2\right)^{\frac{1}{2}} = a + b + \frac{\lambda}{2}$$

$$\Rightarrow \quad a\left(1 + \frac{r_1^2}{a^2}\right)^{\frac{1}{2}} + b\left(1 + \frac{r_1^2}{b^2}\right)^{\frac{1}{2}} = a + b + \frac{\lambda}{2}$$

$$\Rightarrow \quad a\left(1+\frac{r_1^2}{2a^2}\right)+b\left(1+\frac{r_1^2}{2b^2}\right)=a+b+\frac{\lambda}{2}$$

Expanding binomially and neglected higher order term

$$\Rightarrow \quad a+\frac{r_1^2}{2a}+b+\frac{r_1^2}{2b}=a+b+\frac{\lambda}{2}$$

$$\Rightarrow \quad \frac{r_1^2}{2a}+\frac{r_1^2}{2b}=\frac{\lambda}{2} \qquad \Rightarrow \quad \frac{1}{a}+\frac{1}{b}=\frac{\lambda}{r_1^2}$$

$$\Rightarrow \quad \frac{1}{a}+\frac{1}{b}=\frac{\lambda}{\left(r_1^2/\lambda\right)} \qquad \Rightarrow \quad \frac{1}{a}+\frac{1}{b}=\frac{1}{f_1} \qquad \text{where, } f_1=\frac{r_1^2}{\lambda}$$

If the source is at ∞, the distance *a* is ∞.

$$\frac{1}{b}=\frac{1}{f_1} \qquad \text{as, } \frac{1}{a}=0$$

$$\Rightarrow \quad \boxed{f_1=b}$$

Since $f_1=b$, it is also true for converging lens. Therefore, a zone plate behaves like a converging lens.

Let us, consider the next zone to be transparent.

$$SM_2+M_2P=a+b+\lambda$$

$$\Rightarrow \quad \left[\left(SM_0\right)^2+\left(M_0M_2\right)^2\right]^{\frac{1}{2}}+\left[\left(M_0P\right)^2+\left(M_0M_2\right)^2\right]^{\frac{1}{2}}=a+b+\lambda$$

$$\Rightarrow \quad \left(a^2+r_2^2\right)^{\frac{1}{2}}+\left(b^2+r_2^2\right)^{\frac{1}{2}}=a+b+\lambda$$

$$\Rightarrow \quad a\left(1+\frac{r_2^2}{a^2}\right)^{\frac{1}{2}}+b\left(1+\frac{r_2^2}{b^2}\right)^{\frac{1}{2}}=a+b+\lambda$$

$$\Rightarrow \quad a\left(1+\frac{r_2^2}{2a^2}\right)+b\left(1+\frac{r_2^2}{2b^2}\right)=a+b+\lambda$$

Expanding binomially and neglected higher order term

$$\Rightarrow \quad a+\frac{r_2^2}{2a}+b+\frac{r_2^2}{2b}=a+b+\lambda$$

$$\Rightarrow \quad \frac{r_2^2}{2a}+\frac{r_2^2}{2b}=\lambda$$

$$\Rightarrow \quad \frac{1}{a}+\frac{1}{b}=\frac{2\lambda}{r_2^{\,2}} \qquad \Rightarrow \quad \frac{1}{a}+\frac{1}{b}=\frac{1}{\left(\frac{r_2^{\,2}}{2\lambda}\right)}$$

$$\Rightarrow \quad \frac{1}{a}+\frac{1}{b}=\frac{1}{f_2} \qquad \text{where, } f_2=\frac{r_2^{\,2}}{2\lambda}$$

If the source is at ∞, the distance *a* is ∞.

$$\frac{1}{b}=\frac{1}{f_2} \qquad \text{as, } \frac{1}{a}=0$$

$$\Rightarrow \quad \boxed{f_2=b}$$

In general, we can write $\frac{1}{a}+\frac{1}{b}=\frac{1}{f_n}$ where, $\boxed{f_n=\frac{r_n^{\,2}}{n\lambda}}$

$$\boxed{\text{For}, a\to\infty,\ f_n=b}$$

Hence for any zone being transparent, an object at infinity produces a bright image at point P which is at a distance b. Thus, the zone plate behaves as a converging lens.

This focus is known as primary focus of the zone plate. And the corresponding distance from the zone plate to the focus is called the primary focal length.

Multiple Foci of a Zone Plate:

Unlike converging lens, a zone plate has a number of secondary foci in addition to the principal focus or primary focus.

We know that the area of all the half period zones are equal and is given by $\pi b\lambda$.

In zone plate $b=f_1$ so area = $\pi f_1\lambda$.

In order to get maxima or minima at points other than P let us proceed in the following way.

Let, $f_3=\frac{f_1}{2}$ $\qquad \text{Area}=\frac{\pi f_1\lambda}{2}$

Now each zone is divided into two parts and each transparent zone allows two consecutive half period zones of the wave front through it while the next two half period zones are blocked. So, the resultant amplitude $A=(A_1-A_2)+(A_5-A_6)+.....$

Since, $A_1=A_2,\ A_5=A_6 \qquad \Rightarrow A=0\ \Rightarrow I=0$

So, when the focal length is $f_1 = \frac{f_1}{2}$ intensity becomes minimized

Let, $f_3 = \frac{f_1}{3} \quad \Rightarrow \quad \text{Area} = \frac{\pi f_1 \lambda}{3}$

Now each zone is divided into three parts and each transparent zone allows three consecutive half period zone of the wave front through it. While the next three half period zones are blocked.

So, the resultant amplitude, $A = (A_1 - A_2 + A_3) + (A_7 - A_8 + A_9) + ...$

$$\Rightarrow \quad A = \frac{A_1}{2} + \left(\frac{A_1}{2} - A_2 + \frac{A_3}{2}\right) + \frac{A_3}{2} + \frac{A_7}{2} + \left(\frac{A_7}{2} - A_8 + \frac{A_9}{2}\right) + \frac{A_9}{2} + ...$$

$$\Rightarrow \quad A = \frac{1}{2}(A_1 + A_3 + A_7 + A_9 + ...)$$

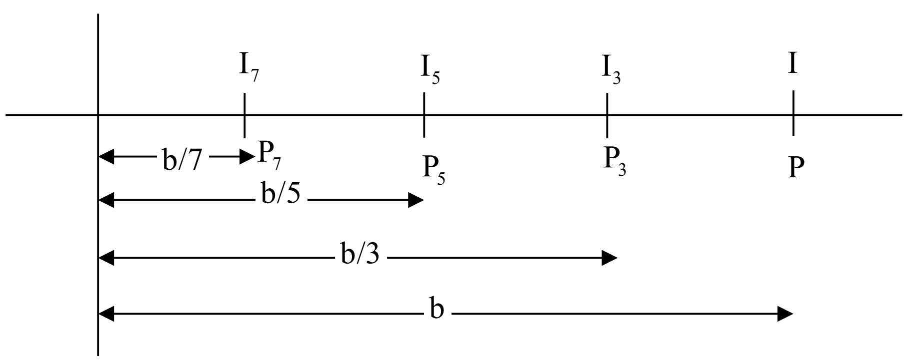
So, intensity is, $I_3 \propto \frac{1}{4}(A_1 + A_3 + A_7 + A_9 + ...)^2 < I_1$

Similarly, we can write,

For, $f_4 = \frac{f_1}{4}, \quad I_4 = 0$

$f_5 = \frac{f_1}{5}, \quad I_5 < I_3 < I_1 \quad I_5 =$

So in general we can write intensity of higher order foci is less than the intensity of lower order foci.



So closer is the foci to the zone plate, lesser is the intensity at the foci.

So, for +ve zone plate the maxima are obtained at $f_1, \frac{f_1}{3}, \frac{f_1}{5}, \frac{f_1}{7}, ...$ with reduced intensity.

Hence a zone plate has multiple foci in contrast with converging lens.

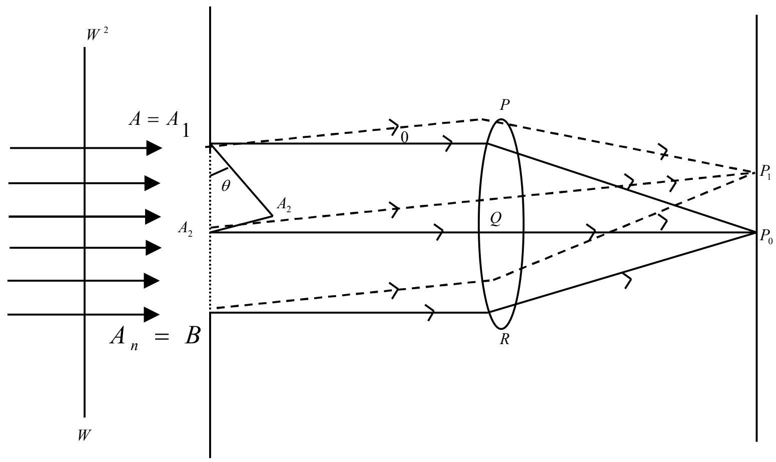
Comparison between zone plate & converging lens

SIMILARITIES	
CONVERGING LENS	**ZONE PLATE**
01. It has a focusing action.	01. It has also a focusing action.
02. The real image is formed on the other side of the source.	02. The real image is also formed on the other side of the source.
03. Focal length depends on the wave length.	03. Focal length also depends upon wave length.
04. The image distance and object distance are connected by the equation $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$	04. The image distance and object distance are connected by the equation $\frac{1}{a}+\frac{1}{b}=\frac{1}{f_n}$, where, $f_n=\frac{r_n^2}{n\lambda}$

DISIMILARITIES	
CONVERGING LENS	**ZONE PLATE**
01. Focusing action is due to refraction of light through the lens.	01. Focusing action is due to diffraction of light through the zone plate.
02. All the light rays refracting through the lens are in same phase.	02. All the secondary wavelet diffracted through the zone plate are not in phase.
03. For monochromatic light a converging lens has a single focus.	03. For monochromatic light a zone plate has multiple foci.
04. The intensity at the focal length is maximum than the zone plate.	04. The intensity at the primary focal length is less as compared to convex lens.
05. In convex lens, focal length of violet light is less than that of red light, $i.e\ \ f_v < f_R$.	05. In case of zone plate, focal length of violet light is more than that of red light. $i.e\ \ f_v > f_R$.
06. A convex lens converges the light.	06. A zone plate can converge and diverge the light.
07. A convex lens can't act like a concave lens in a single medium.	07. A zone plate can act like a concave lens.
08. No virtual focus on the side of the source.	08. It has virtual foci on the side of the source.

Fraunhofer's diffraction due to a single slit

The fig. represents a section AB of narrow slit of width 'b' perpendicular to the place of the paper. Let a plane wave-front ww[1] of monochromatic light of wavelength 'λ' propagating normally to the slit be incident on it. The diffracted light is focused by means of a convert lens on a screen placed in the focal plane of the lens.



According to Huygens principle, every point of the wave front in the place of the site is a source of secondary spherical wavelets, which spread out to the right in all direction. The secondary wavelets traveling normally to the slit are brought to focus at 'p_o' by the lens. Thus 'p_o' is a bright central image.

The secondary wavelets traveling at an angle 'o' with the normal are focused at point 'p_1' on the screen. The intensity of the point 'p_1' (maximum or minimum) depends upon the path difference between the secondary wavelets originating from corresponding points of the wave front.

Let the width of the slit is divided into n. equal parts A_1, A_2, A_3...,

Each has same amplitude. Let 'Δ' be the distance between two consecutive points. Let $A_2A_2^1$ be a perpendicular drawn from A_1(= A) to A_2P_1.

The path difference between secondary wavelets from A_1& A_2 in direction 'θ'.

$$A_2A_2^1 = A_1A_2.\sin\theta = \Delta\sin\theta \qquad \ldots\ldots. (1)$$

The corresponding phase difference,

$$\varphi - \frac{2\pi}{\lambda} \times pathdifference = \frac{2\pi}{\lambda} \times \Delta\sin\theta \qquad \ldots\ldots. (2)$$

If the field at point 'P_1' due to disturbances emanating from point 'A_1' is "a Cos θ" then the field due to the disturbance emanating from point 'A_2' would be $a.Cos(wt-\varphi)$and so on.

Thus, the resultant field at point ‘p’

$$R = a[\cos wt + \cos(wt - \varphi) + \cos(wt - 2\varphi) + \cdots \ldots \ldots \ldots . + \cos\{wt - (n-1)\varphi\}]$$

$$Where, \quad \varphi = \frac{2\pi}{\lambda} \triangleleft \sin\theta$$

$$\therefore R = a\left\{\frac{\sin\left(\frac{n\varphi}{2}\right)}{\sin\left(\varphi/_2\right)}\right\}.\cos\left[wt - \frac{n\varphi}{2}\right]$$

$$\Rightarrow R = E_\theta.\cos\left[wt - \frac{n\varphi}{2}\right] \ldots\ldots\ldots(3)$$

$$Where, \ E_\theta = a.\left\{\frac{\sin\left(n\varphi/_2\right)}{\sin \varphi/_2}\right\} \quad \ldots\ldots\ldots (4)$$

‘E_θ’ is the amplitude of resultant field.

In the limiting case when, $n \to \infty, \Delta \to 0, \ \varphi \to 0 \ \ and \ \ n\Delta \to b$.

Then, $$\frac{n\varphi}{2} = \frac{n}{2} \times \frac{2\pi}{\lambda} \Delta \sin\theta = \frac{\pi}{\lambda} \times (n\Delta \sin\theta) = \frac{\pi}{\lambda} \times b \sin\theta \quad \ldots\ldots (5)$$

Further, $$\frac{\varphi}{2} = \frac{1}{2}.\frac{2\pi}{\lambda} \times \Delta \sin\theta = \frac{\pi}{\lambda}.\frac{n\Delta \sin\theta}{n} = \frac{\pi}{\lambda} \times \frac{b \sin\theta}{n} \quad \ldots\ldots\ldots\ldots (6)$$

$$\sin\left(\frac{\varphi}{2}\right) \approx \left(\frac{\varphi}{2}\right).$$

Hence, $$E_\theta = a\left\{\frac{\sin\left(\frac{\pi b \sin\theta}{\lambda}\right)}{\left(\frac{\pi}{\lambda}.\frac{b \sin\theta}{n}\right)}\right\} \quad \Rightarrow \quad E_\theta = na.\left(\frac{\sin\beta}{\beta}\right)$$

$$\Rightarrow E_\theta = A.\left(\frac{\sin\beta}{\beta}\right) \ldots \ldots \ldots \ldots \ldots . (7)$$

Where, $$A = na \ . \ \& \ \ \beta = \left(\frac{\pi b \sin\theta}{\lambda}\right) \qquad (08)$$

$$\therefore R = E_\theta.\cos(wt - \beta) \ \ldots \ldots \ldots \ldots (9)$$

∴ Intensity of resultant disturbance.

$$I = (E_\theta)^2 = A^2.\left(\frac{\sin^2\beta}{\beta^2}\right) \Rightarrow I = I_O\left(\frac{\sin^2\beta}{\beta^2}\right) \quad \ldots \ldots \ldots .. \ldots \ldots . (10)$$

Where, $I_O = A^2$ represents intensity at θ = 0.

Position of maxima and minima

Minima: It is clear from equation (10) that the intensity is zero.

When $\quad \sin\beta = 0 \quad \Rightarrow \quad \beta = m\pi, \ \ where m = \pm1, \pm2, \pm3.........$

[But m ≠ 0, because, when m = 0, β = 0 and hence $\left(\frac{\sin^2\beta}{\beta^2}\right)$ = I & hence $I = I_O$ which corresponds to maximum of intensity].

When, $\quad \beta = m\pi \qquad \Rightarrow \frac{\pi b \sin\theta}{\lambda} = m\pi \quad \Rightarrow b\sin\theta = m\lambda \quad \dots\dots\dots(11)$

$$Where, m = \pm1, \pm2, \pm3, ...$$

This is the required condition for minima. It is clean from equation (11) that, first minima occurs at $\theta = \sin^{-1}\left(\frac{\lambda}{b}\right)$, second minima occurs at $\theta = \sin^{-1}\left(\frac{2\lambda}{b}\right)$ and so on.

Maxima: To obtain the condition for maxima, let us differentiate equation (10) with respect to 'β' and equate it to zero.

$$\frac{dI}{d\beta} = 0$$

$$\Rightarrow I_O.\frac{d}{d\beta}\left(\frac{\sin\beta}{\beta}\right)^2 = 0$$

$$\Rightarrow I_O.2\left(\frac{\sin\beta}{\beta}\right).\left(\frac{\beta Cos\beta - \sin\beta}{\beta^2}\right) = 0.$$

$$\Rightarrow \frac{\beta Cos\beta - \sin\beta}{\beta^2} = 0$$

$$\Rightarrow \beta Cos\beta - \sin\beta = 0$$

$$\Rightarrow \beta = \tan\beta \quad \dots\dots\dots\dots(12)$$

Equation (12) can be solved graphically by plotting graphics

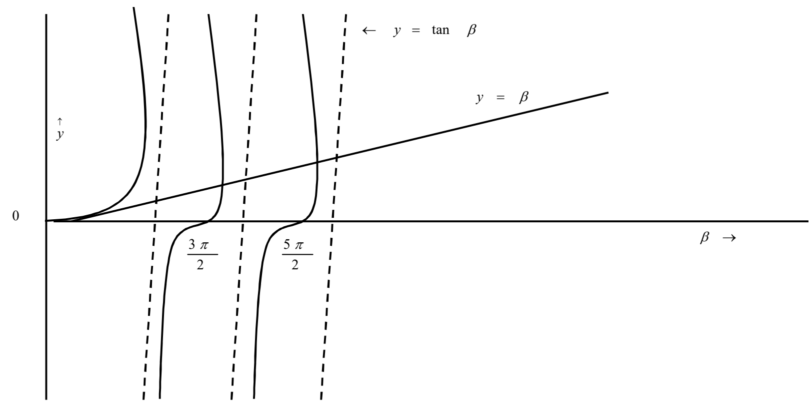
$$y = \beta..............(A) \qquad \& \qquad y = \tan\beta..............(B)$$

Curve (A) is a straight line passing through origin and makes an angle 45° with x-axis.

Curve (B) is a discontinuous curve having a number of branches. The point of intersection of these two curves gives values of 'β' satisfying equation (12).

The values of 'β' approximately are $\beta = 0, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, ...$

Substituting the approximately values of 'β' in equation (10) we get the intensities of various maxima.



Intensity of central \ principal maximum

$$I_O = A^2.\left(\frac{sin\,o}{o}\right)^2 = A^2.$$

Clearly most of the incident light concentrated in the principal maximum which occurs in the direction given by

$$\beta = o \quad \Rightarrow \frac{\pi b\, sin\,\theta}{\lambda} = o \quad \Rightarrow \theta = 0.$$

i.e. in the same direction as incident light.

Thus, the diffraction pattern consists of a bright principal maximum in the direction of incident light having alternative minima and week subsidiary maxima of rapidly decreasing intensity on either side of it.